Gaussian elimination with partial pivoting.

1. Perform Gaussian elimination with partial pivoting on this matrix.

$$\left(\begin{array}{rrr|r} 1.5 & 4.6 & -0.8 & 12.3 \\ -3.5 & -4.6 & 4.8 & -22.3 \\ 5.0 & 2.0 & 4.0 & 1.0 \end{array}\right)$$

Answer:

$$\left(\begin{array}{rrr|r} 5 & 2 & 4 & 1 \\ 0 & 4 & -2 & 12 \\ 0 & 0 & 6 & -12 \end{array}\right)$$

2. Perform Gaussian elimination with partial pivoting on this matrix.

$$\left(\begin{array}{rr|r} 4.1 & 8.55 & 17.45 \\ 8.2 & 4.5 & -2.9 \end{array}\right)$$

Answer:

$$\left(\begin{array}{rr|r} 8.2 & 4.5 & -2.9 \\ 0 & 6.3 & 18.9 \end{array}\right)$$

3. Perform Gaussian elimination with partial pivoting on this matrix.

$$\left(\begin{array}{rrrr|r} -3.5 & -7.4 & -0.1 & 3.7 & 22.3 \\ 4.0 & -2.0 & 1.5 & -6.4 & -9.7 \\ 5.0 & 2.0 & 3.0 & -1.0 & 1.0 \\ 4.5 & 4.2 & 8.9 & -4.1 & -7.3 \end{array}\right)$$

Answer:

$$\left(\begin{array}{rrrr|r} 5 & 2 & 3 & -1 & 1 \\ 0 & -6 & 2 & 3 & 23 \\ 0 & 0 & 7 & -2 & 1 \\ 0 & 0 & 0 & -8 & -24 \end{array}\right)$$

4. If a matrix is diagonally dominant, is there ever any need for partial pivoting during the Gaussian elimination algorithm?

Answer: No. If the diagonal is already greater than the sum of the absolute values of the off-diagonal entries of the corresponding rows and columns, then adding scalar multiples of one row onto another where the scalar multiples are never greater than 1 in absolute value will never result in a need for partial pivoting.

5. What is the run time of Gaussian elimination with partial pivoting? What is the run time of Gaussian elimination without partial pivoting?

Answer: They are both $O(n^3)$ for solving a system of n linear equations in n unknowns.